Simplification of Switching Function

Agenda

- Simplification Goals
- Karnaugh Maps
- Plotting Function in Canonical Form on the K-map
- Simplification Using K-Map
- POS Form Using K-Map
- Quine-McCluskey Tubular Minimization Method

Simplification Goals

- Minimize the cost of realizing a function.
- Minimize the number of circuit element.
- Minimize the number of literals in each term.

Determine the number of terms and literals in the following functions:

$$g(A,B,C) = A\bar{B} + \bar{A}B + AC$$

$$f(X,Y,Z) = \bar{X}Y(Z + \bar{Y}X) + \bar{Y}Z$$

Simplification Goals

Use switching algebra to find minimal SOP and POS forms for the function $f(X,Y,Z)$ of Example 3.1.

This expression can be minimized as follows.

$$\begin{aligned} f(X,Y,Z) &= \bar{X}Y(Z+\bar{Y}X)+\bar{Y}Z & \\ &= \bar{X}YZ+\bar{X}Y\bar{Y}X+\bar{Y}Z & \text{[P5(b)]} \\ &= \bar{X}YZ+\bar{Y}Z & \text{[P6(b), P2(a)]} \\ &= \bar{X}Z+\bar{Y}Z & \text{[T7(a)]} \\ &= (\bar{X}+\bar{Y})Z & \text{[P5(b)]} \end{aligned}$$

Simplification Goals

Use switching algebra to find a minimal SOP expression for the function

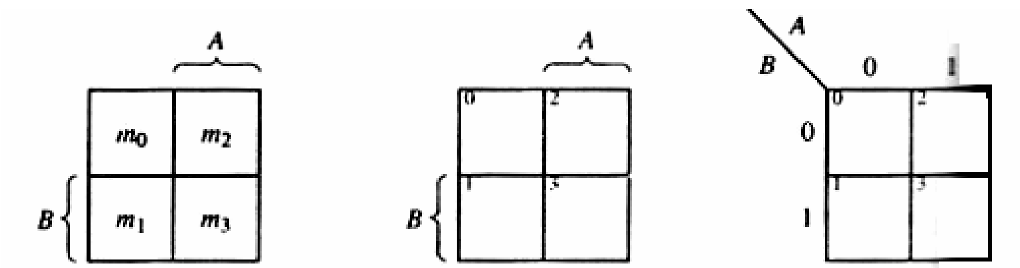
$$f(A,B,C,D) = ABC + ABD + \bar{A}B\bar{C} + CD + B\bar{D}$$

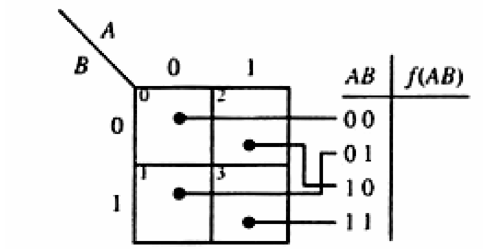
which has four variables and 13 literals.

$$
\begin{aligned}
f(A,B,C,D) &= ABC + ABD + \bar{A}B\bar{C} + CD + B\bar{D} & \\
&= ABC + AB + \bar{A}B\bar{C} + CD + B\bar{D} & \text{[T7(a)]} \\
&= ABC + AB + B\bar{C} + CD + B\bar{D} & \text{[T7(a)]} \\
&= AB + B\bar{C} + CD + B\bar{D} & \text{[T4(a)]} \\
&= AB + CD + B(\bar{C} + \bar{D}) & \text{[P5(b)]} \\
&= AB + CD + B\overline{CD} & \text{[T8(b)]} \\
&= AB + CD + B & \text{[T5(a)]} \\
&= B + CD & \text{[T4(a)]}
\end{aligned}
$$

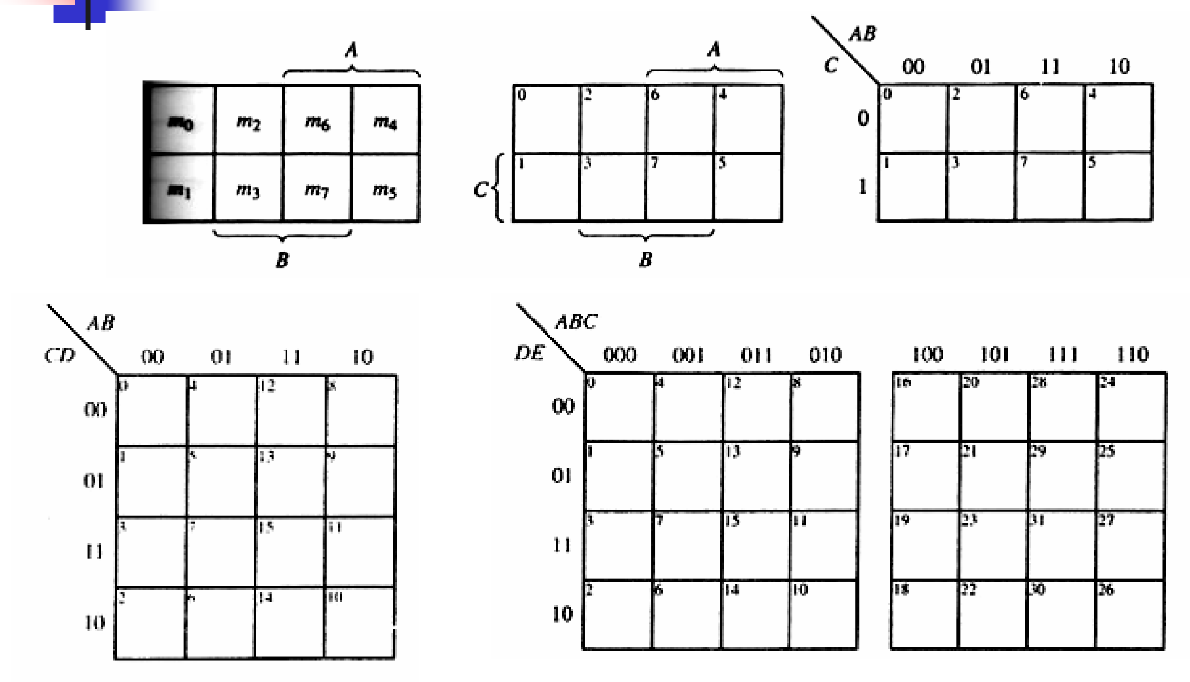
Note that the number of literals has been reduced from 13 to 3.

Karnaugh Maps

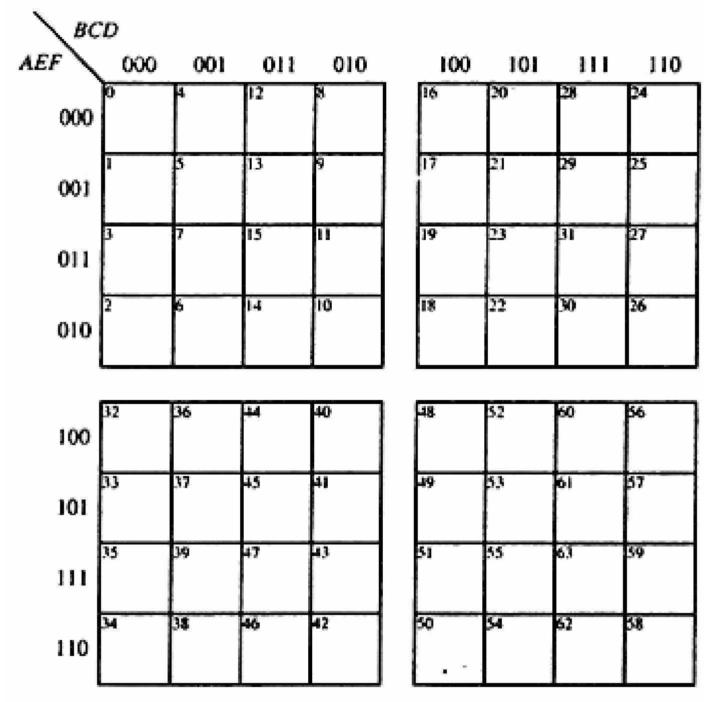




Karnaugh Maps

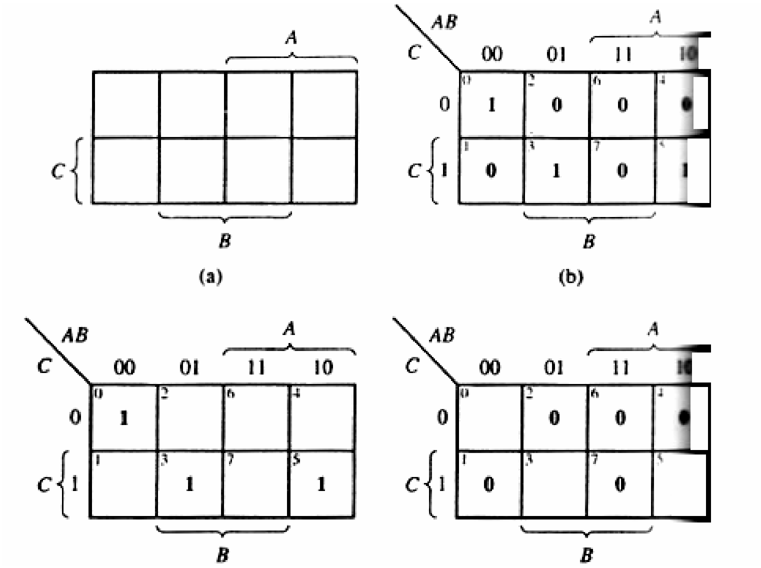


Karnaugh Maps



Plotting Functions in Canonical Form on the K-Map

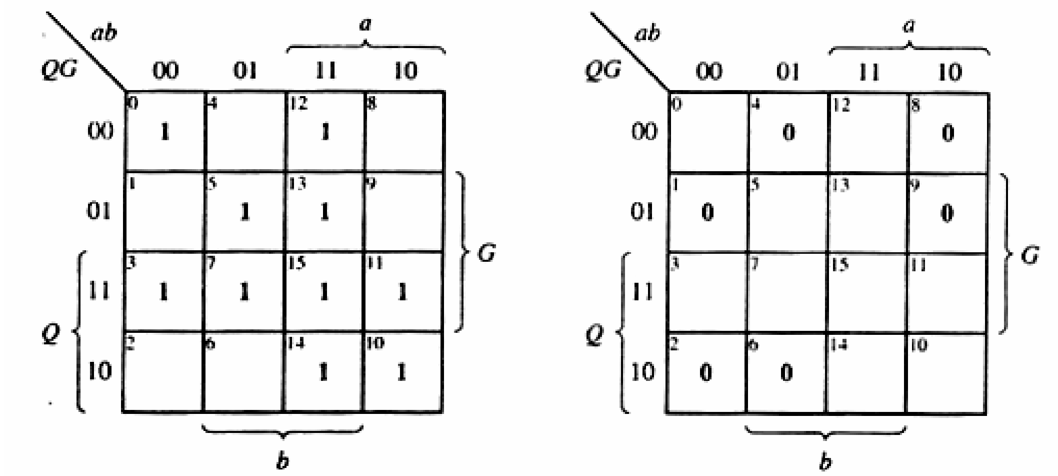
$$f(A, B, C) = m(0, 3, 5) = m_0 + m_3 + m_5$$
$$= \prod M(1, 2, 4, 6, 7) = M_1 M_2 M_4 M_6 M_7$$



Plotting Functions in Canonical Form on the K-Map

Let us plot the following function on a K-map:

$$f(a,b,Q,G) = m(0,3,5,7,10,11,12,13,14,15)$$
$$= \prod M(1,2,4,6,8,9)$$



Plotting Functions in Canonical Form on the K-Map

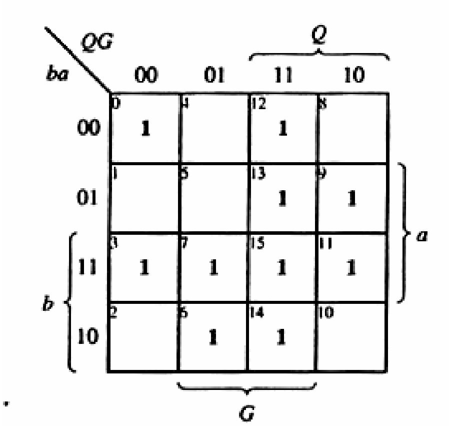
Let us repeat Example 3.4 with the variables reordered to give $f(Q,G,b,a)$.

First, write the minterms of $f(a,b,Q,G)$:

$$f(a,b,Q,G) = \sum m(0,3,5,7,10,11,12,13,14,15)$$
$$= \bar{a}\bar{b}\bar{Q}\bar{G} + \bar{a}\bar{b}QG + \bar{a}b\bar{Q}G + \bar{a}bQG + a\bar{b}Q\bar{G}$$
$$+ a\bar{b}QG + ab\bar{Q}\bar{G} + ab\bar{Q}G + abQ\bar{G} + abQG$$

Next, rearrange the variables:

$$f(Q,G,b,a) = \bar{Q}\bar{G}\bar{b}\bar{a} + QG\bar{b}\bar{a} + \bar{Q}Gb\bar{a} + QGb\bar{a} + Q\bar{G}\bar{b}a$$
$$+ QG\bar{b}a + \bar{Q}\bar{G}ba + \bar{Q}Gba + Q\bar{G}ba + QGba$$
$$= \sum m(0,12,6,14,9,13,3,7,11,15)$$
$$= \sum m(0,3,6,7,9,11,12,13,14,15)$$

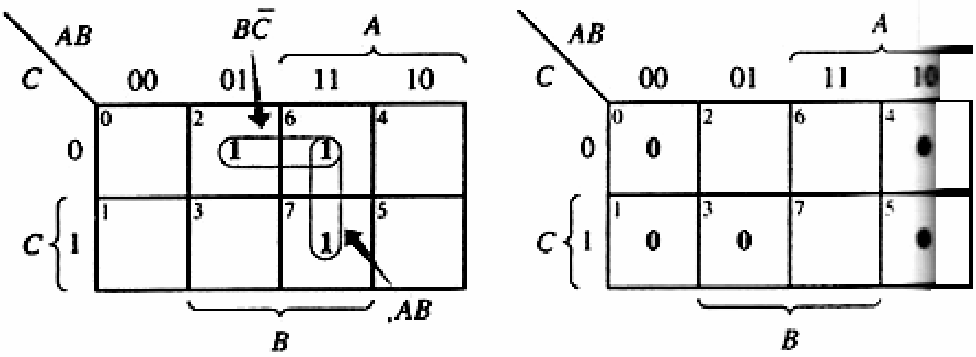


Plotting Functions in Canonical Form on the K-Map

Consider the following function, which is expressed as a sum of products.

$$f(A,B,C) = AB + B\bar{C}$$

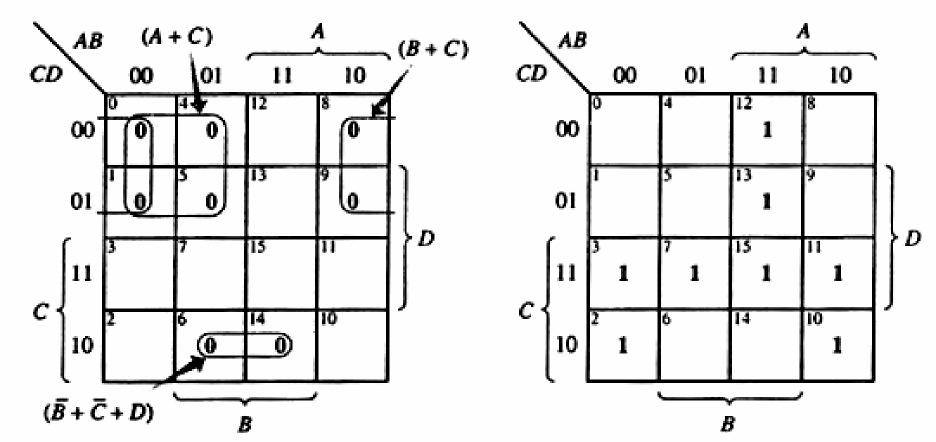
We wish to plot the function on a K-map and determine its minterm and maxterm lists.



Plotting Functions in Canonical Form on the K-Map

Let us plot the following function on the K-map and determine its minterm and maxterm lists.

$$f(A,B,C,D) = (A + C)(B + C)(\bar{B} + \bar{C} + D)$$



Plotting Functions in Canonical Form on the K-Map

Derive the minterm list of the function

$$f(A,B,C,D) = (\bar{A} + \bar{B})(\bar{A} + C + \bar{D})(\bar{B} + \bar{C} + \bar{D})$$

We begin by complementing the function and applying DeMorgan's

$$\bar{f}(A, B, C, D) = \overline{(\bar{A} + \bar{B})(\bar{A} + C + \bar{D})(\bar{B} + \bar{C} + \bar{D})}$$

$$= \overline{(\bar{A} + \bar{B})} + \overline{(\bar{A} + C + \bar{D})} + \overline{(\bar{B} + \bar{C} + \bar{D})}$$

$$= AB + A\bar{C}D + BCD$$

$\bar{f}(A, B, C, D) = AB + A\bar{C}D + BCD$ is plotted on the K-map

$$\bar{f}(A, B, C, D) = \sum m(7, 9, 12, 13, 14, 15)$$

$$f(A, B, C, D) = \sum m(0, 1, 2, 3, 4, 5, 6, 8, 10, 11)$$

Plotting Functions in Canonical Form on the K-Map

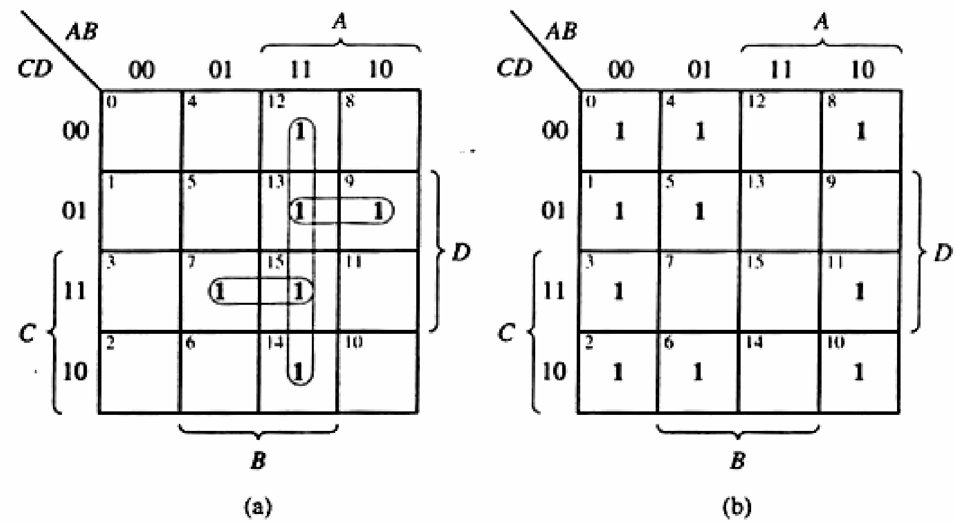
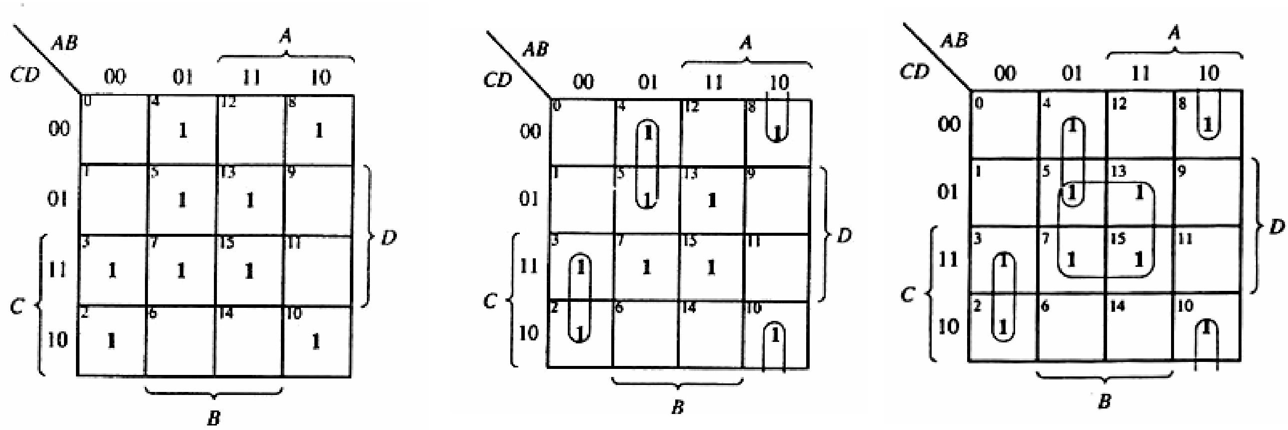


Figure 3.9 K-maps for Example 3.8. **(a)** K-map of $\bar{f}(A, B, C, D)$. **(b)** Corresponding K-map of $f(A, B, C, D)$.

Simplification of Switching Function Using K-Maps

simplify the following function.

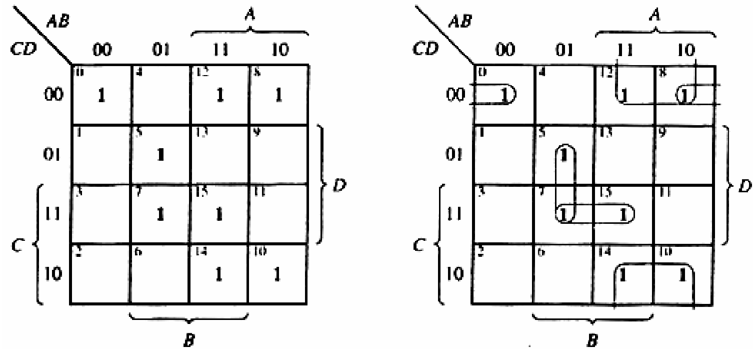
$$f(A,B,C,D) = \sum m(2,3,4,5,7,8,10,13,15)$$

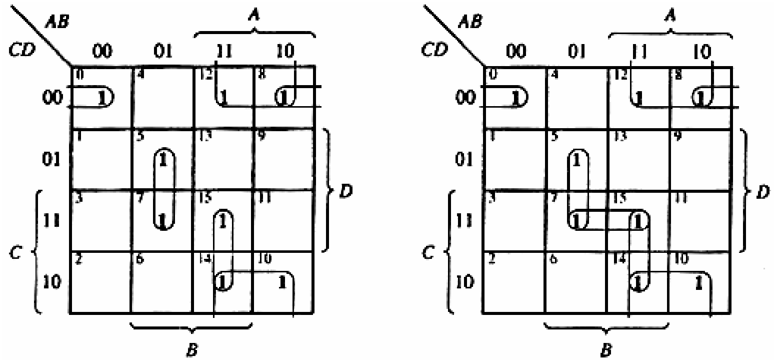


Simplification of Switching Function Using K-Maps

Let us use the K-map to simplify the following function.

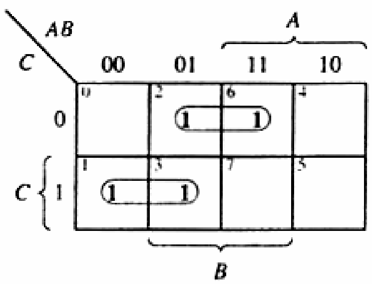
$$f(A,B,C,D)=\sum m(0,5,7,8,10,12,14,15)$$





Simplification of Switching Function Using K-Maps

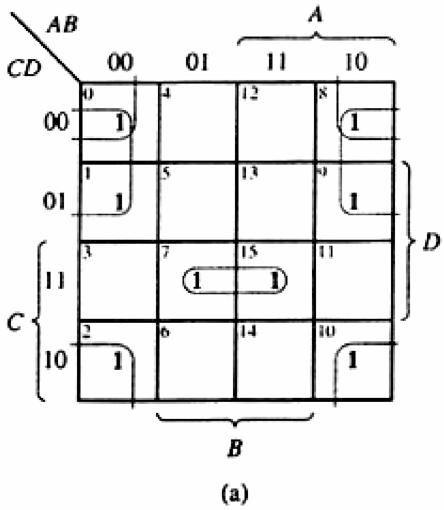
Find a minimum SOP expression for $f(A,B,C)=\sum m(1,2,3,6)$.



$$f(A,B,C)=\bar{A}C+B\bar{C}$$

Simplification of Switching Function Using K-Maps

Find a minimum SOP expression for
$f(A,B,C,D) = \sum m(0,1,2,7,8,9,10,15)$.

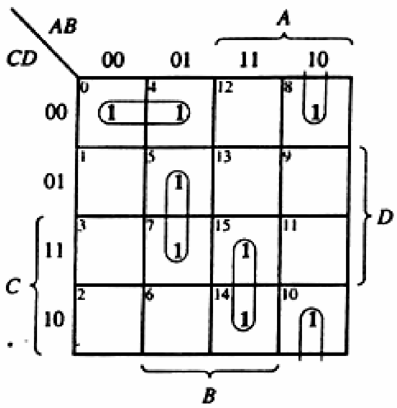


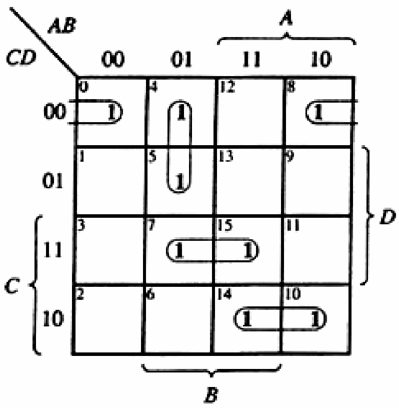
(a)

Figure 3.16 $f(A,B,C,D) = \bar{B}\bar{D} + \bar{B}\bar{C} + BCD$.

Simplification of Switching Function Using K-Maps

Find a minimum SOP expression for
$f(A,B,C,D) = \sum m(0,4,5,7,8,10,14,15)$.





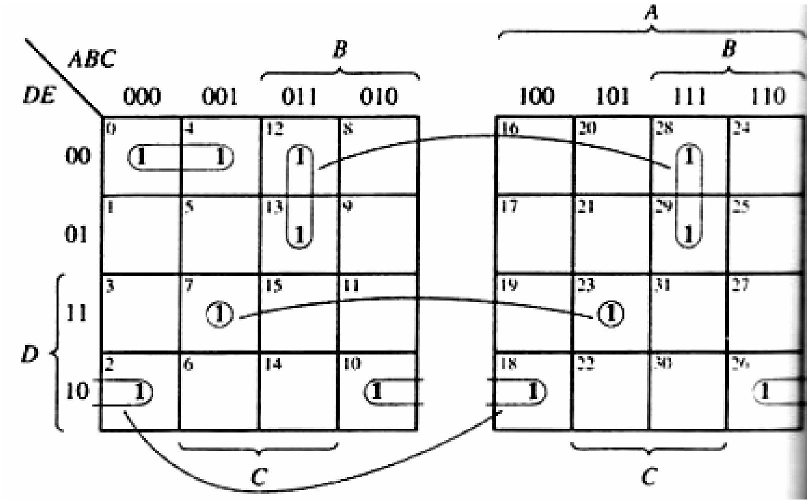
$f(A,B,C,D) = \bar{A}\bar{C}\bar{D} + \bar{A}BD + ABC + A\bar{B}\bar{D}$

$f(A,B,C,D) = \bar{B}\bar{C}\bar{D} + \bar{A}B\bar{C} + BCD + AC\bar{D}$

Simplification of Switching Function Using K-Maps

Find a minimum SOP expression for

$$f(A,B,C,D,E) = \sum m(0,2,4,7,10,12,13,18,23,26,28,29)$$

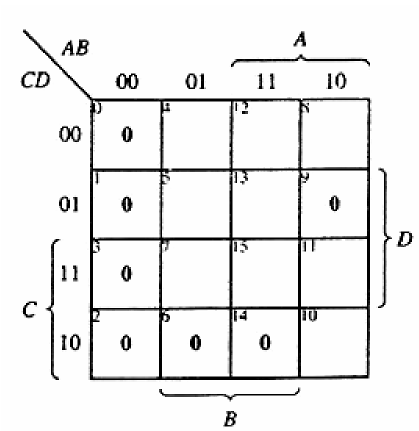


$$f(A,B,C,D,E) = \bar{A}\bar{B}\bar{D}\bar{E} + BC\bar{D} + \bar{B}CDE + \bar{C}D\bar{E}$$

POS Form Using K-Maps

Let us find the minimum POS form for the function

$$f(A,B,C,D) = \prod M(0,1,2,3,6,9,14)$$



$$f(A,B,C,D) = (A+B)(B+C+\bar{D})(\bar{B}+\bar{C}+D)$$

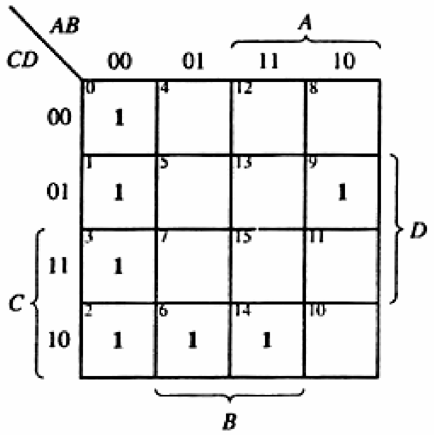
POS Form Using K-Maps

Repeat Example

Given the function

$$f(A,B,C,D) = \prod M(0,1,2,3,6,9,14)$$

$$\bar{f}(A,B,C,D) = \sum m(0,1,2,3,6,9,14)$$



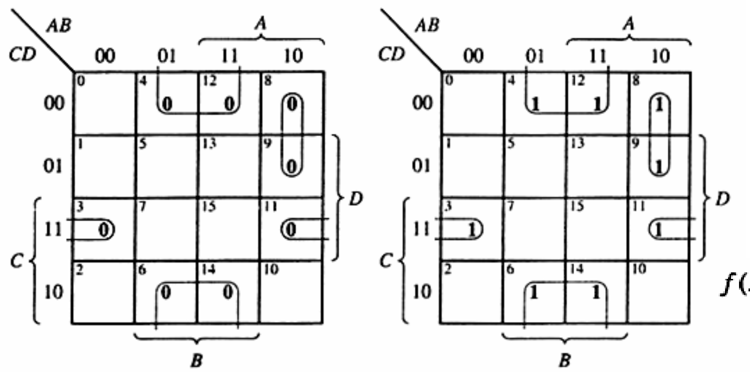
(a)

$$\bar{f}(A,B,C,D) = \bar{A}\bar{B} + \bar{B}\bar{C}D + BC\bar{D}$$

$$\begin{aligned} f(A,B,C,D) &= \overline{\bar{A}\bar{B} + \bar{B}\bar{C}D + BC\bar{D}} \\ &= (\overline{\bar{A}\bar{B}})(\overline{\bar{B}\bar{C}D})(\overline{BC\bar{D}}) \\ &= (A+B)(B+C+\bar{D})(\bar{B}+\bar{C}+D) \end{aligned}$$

POS Form Using K-Maps

Find a minimum POS expression for
$f(A,B,C,D) = \prod M(3,4,6,8,9,11,12,14)$.



$$\bar{f}(A,B,C,D) = B\bar{D} + \bar{B}CD + A\bar{B}\bar{C}$$

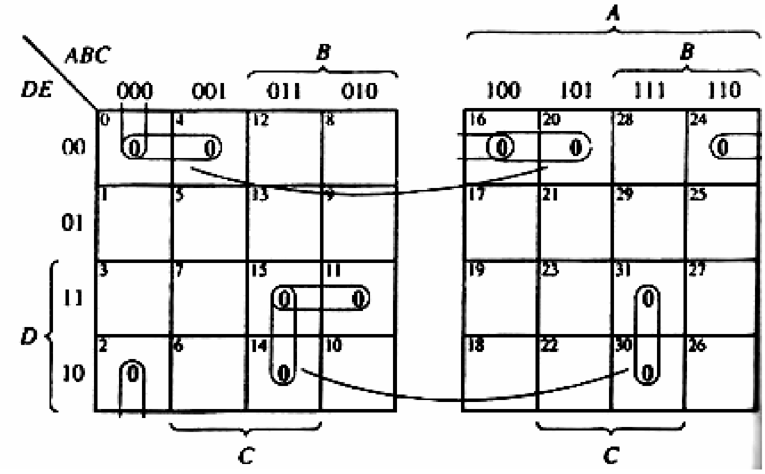
$$\begin{aligned} f(A,B,C,D) &= \overline{B\bar{D} + \bar{B}CD + A\bar{B}\bar{C}} \\ &= (\overline{B\bar{D}})(\overline{\bar{B}CD})(\overline{A\bar{B}\bar{C}}) \\ &= (\bar{B}+D)(B+\bar{C}+\bar{D})(\bar{A}+B+C) \end{aligned}$$

$$f(A,B,C,D) = (\bar{B}+D)(B+\bar{C}+\bar{D})(\bar{A}+B+C)$$

POS Form Using K-Maps

Derive a minimum POS expression for the function

$$f(A,B,C,D,E) = \prod M(0,2,4,11,14,15,16,20,24,30,31)$$

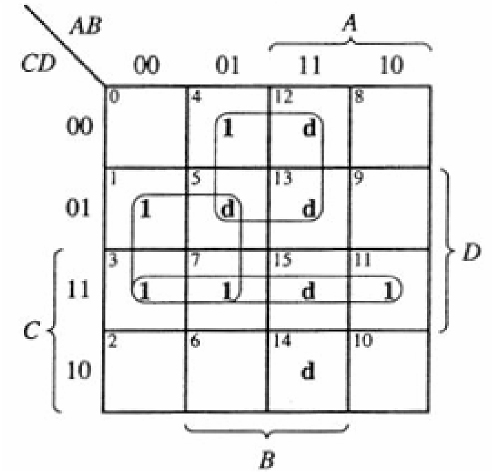


$f(A,B,C,D,E) =$

POS Form Using K-Maps

We wish to minimize the following function in both SOP and POS forms using K-maps.

$$f(A,B,C,D) = \sum m(1,3,4,7,11) + d(5,12,13,14,15)$$
$$= \prod M(0,2,6,8,9,10) \cdot D(5,12,13,14,15)$$



$f(A,B,C,D) = B\bar{C} + \bar{A}D + CD$

$f(A,B,C,D) = (B+D)(\bar{C}+D)(\bar{A}+C)$

Quine-McCluskey Tabular Minimization Method

- Step for Q-M Method is following.
  - STEP 1: Partition each term into groups according the number of 1 bits in their binary representation.
  - STEP 2: Perform a term between neighboring group (n and n+1), two term must differ in exactly one literal.
  - STEP 3:Eliminated variable that be change.

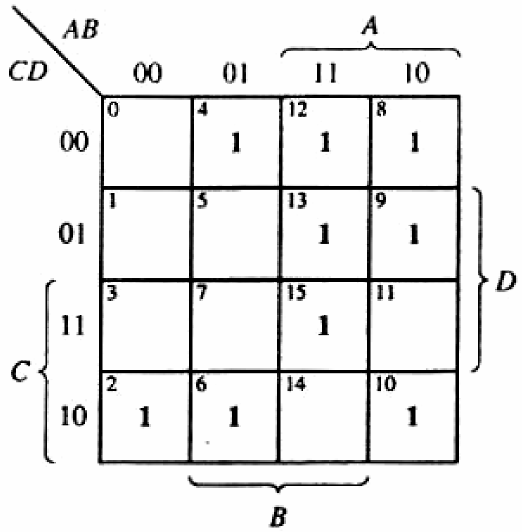
Quine-McCluskey Tabular Minimization Method

- Step for Q-M Method is following.
  - STEP 4: Repeat Step 2,3 Until it is not grouping by a larger implicant.
  - STEP 5: Select a minimum number of prime implicant the cover all the term of the switching function.

Quine-McCluskey Tabular Minimization Method

Let us use the Q–M technique to minimize the function

$$f(A,B,C,D)=\sum m(2,4,6,8,9,10,12,13,15)$$



Minterms	$ABCD$	
2	0010	Group 1 (a single 1)
4	0100	
8	1000	
6	0110	Group 2 (two 1's)
9	1001	
10	1010	
12	1100	
13	1101	Group 3 (three 1's)
15	1111	Group 4 (four 1's)

Quine-McCluskey Tabular Minimization Method

List 1			List 2			List 3		
Minterm	$ABCD$		Minterms	$ABCD$		Minterms	$ABCD$	
2	0010	✓	2, 6	0-10	PI_2	8, 9, 12, 13	1-0-	PI_1
4	0100	✓	2, 10	-010	PI_3			
8	1000	✓	4, 6	01-0	PI_4			
6	0110	✓	4, 12	-100	PI_5			
9	1001	✓	8, 9	100-	✓			
10	1010	✓	8, 10	10-0	PI_6			
12	1100	✓	8, 12	1-00	✓			
13	1101	✓	9, 13	1-01	✓			
15	1111	✓	12, 13	110-	✓			
			13, 15	11-1	PI_7			

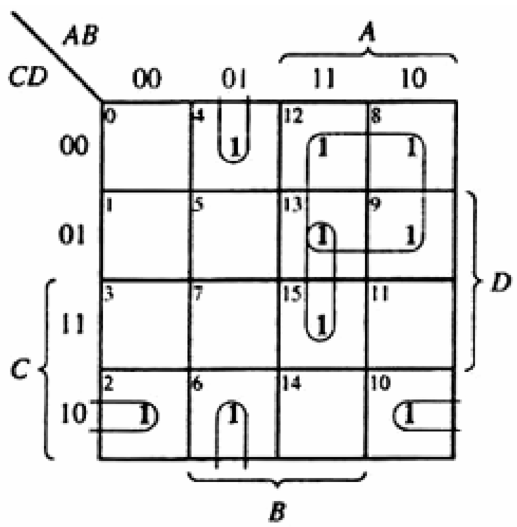
Quine-McCluskey Tabular Minimization Method

	2	4	6	8 √	9 √	10	12 √	13 √	15 √
•• PI_1				×	⊗		×	×	
PI_2	×		×						
PI_3	×					×			
PI_4		×	×						
PI_5		×					×		
PI_6				×		×			
•• PI_7								×	⊗

	2 √	4 √	6 √	10 √
PI_2	×		×	
• PI_3	×			×
• PI_4		×	×	
PI_5		×		
PI_6				×

Quine-McCluskey Tabular Minimization Method

$$
\begin{aligned}
f(A,B,C,D) &= PI_1 + PI_3 + PI_4 + PI_7 \\
&= \text{1-0-} + \text{-010} + \text{01-0} + \text{11-1} \\
&= A\bar{C} + \bar{B}C\bar{D} + \bar{A}B\bar{D} + ABD
\end{aligned}
$$



Quine-McCluskey Tabular Minimization Method

$$f(A, B, C, D) = \sum m(0, 1, 5, 6, 7, 8, 9, 10, 11, 13, 14, 15)$$

Quine-McCluskey Tabular Minimization Method

	√ 0	√ 1	5	√ 6	√ 7	√ 8	√ 9	10	11	13	√ 14	√ 15
• • PI_1	⊗	×				×	×					
PI_2		×	×				×			×		
PI_3			×		×					×		×
PI_4						×	×	×	×			
PI_5							×		×	×		×
PI_6								×	×		×	×
• • PI_7				⊗	×						×	×

	5	10	11	13
PI_2	×			×
PI_3	×			×
PI_4		×	×	
PI_5			×	×
PI_6		×	×	

	√ 5	√ 10
• PI_2	×	
• PI_4		×

Quine-McCluskey Tabular Minimization Method

$$f(A, B, C) = \sum m(1, 2, 3, 4, 5, 6)$$

Quine-McCluskey Tabular Minimization Method

	√ 1	2	√ 3	4	5	6
•PI_1	×		×			
PI_2		×	×			
PI_3		×				×
PI_4				×		×
PI_5				×	×	
PI_6	×				×	

	2	4	5	6
PI_2	×			
PI_3	×			×
PI_4		×		×
PI_5		×	×	
PI_6			×	

	√ 2	√ 4	√ 5	√ 6
•PI_3	×			×
PI_4		×		×
•PI_5		×	×	

Quine-McCluskey Tabular Minimization Method

We want to use the Q–M approach to minimize the function

$$f(A,B,C,D,E) = m(2,3,7,10,12,15,27) + d(5,18,19,21,23)$$

List 1

Minterm	ABCDE	
2	00010	✓
3	00011	✓
5	00101	✓
10	01010	✓
12	01100	PI_7
18	10010	✓
7	00111	✓
19	10011	✓
21	10101	✓
15	01111	✓
23	10111	✓
27	11011	✓

List 2

Minterms	ABCDE	
2, 3	0001-	✓
2, 10	0-010	PI_4
2, 18	-0010	✓
3, 7	00-11	✓
3, 19	-0011	✓
5, 7	001-1	✓
5, 21	-0101	✓
18, 19	1001-	✓
7, 15	0-111	PI_5
7, 23	-0111	✓
19, 23	10-11	✓
19, 27	1-011	PI_6
21, 23	101-1	✓

List 3

Minterms	ABCDE	
2, 3, 18, 19	-001-	PI_1
3, 7, 19, 23	-0-11	PI_2
5, 7, 21, 23	-01-1	PI_3

Quine-McCluskey Tabular Minimization Method

$$f(A, B, C, D, E) = PI_1 + PI_4 + PI_5 + PI_6 + PI_7$$

$$f(A, B, C, D, E) = PI_2 + PI_4 + PI_5 + PI_6 + PI_7$$

	√ 2·	3	√ 7	√ 10	√ 12	√ 15	√ 27
PI_1	×	×					
PI_2		×	×				
PI_3			×				
•• PI_4	×			⊗			
•• PI_5			×			⊗	
•• PI_6							⊗
•• PI_7					⊗		

Let us use the tabular method to obtain a minimum realization for the functions

$$f_\alpha(A,B,C,D) = \sum m(0,2,7,10) + d(12,15)$$
$$f_\beta(A,B,C,D) = \sum m(2,4,5) + d(6,7,8,10)$$
$$f_\gamma(A,B,C,D) = \sum m(2,7,8) + d(0,5,13)$$

Min term	List 1 ABCD	Flags		Min terms	List 2 ABCD	Flags		Min terms	List 3 ABCD	Fl
0	0000	$\alpha\gamma$	✓	0, 2	00-0	$\alpha\gamma$	PI_2	4, 5, 6, 7	01--	β
2	0010	$\alpha\beta\gamma$	PI_{10}	0, 8	-000	γ	PI_3			
4	0100	β	✓	2, 6	0-10	β	PI_4			
8	1000	$\beta\gamma$	PI_{11}	2, 10	-010	$\alpha\beta$	PI_5			
5	0101	$\beta\gamma$	✓	4, 5	010-	β	✓			
6	0110	β	✓	4, 6	01-0	β	✓			
10	1010	$\alpha\beta$	✓	8, 10	10-0	β	PI_6			
12	1100	α	PI_{12}	5, 7	01-1	$\beta\gamma$	PI_7			
7	0111	$\alpha\beta\gamma$	PI_{13}	5, 13	-101	γ	PI_8			
13	1101	γ	✓	6, 7	011-	β	✓			
15	1111	α	✓	7, 15	-111	α	PI_9			

Quine-McCluskey Tabular Minimization Method

	f_α				f_β			f_γ		
	√	√		√	√	√	√	√		
	0	2	7	10	2	4	5	2	7	8
••PI_1 β						⊗	×			
••PI_2 $\alpha\gamma$	⊗	×						×		
PI_3 γ										×
PI_4 β					×					
••PI_5 $\alpha\beta$		×		⊗	×					
PI_6 β										
PI_7 $\beta\gamma$							×		×	
PI_8 γ										
PI_9 α			×							
PI_{10} $\alpha\beta\gamma$		×			×			×		
PI_{11} $\beta\gamma$										×
PI_{12} α										
PI_{13} $\alpha\beta\gamma$			×						×	

	f_α	f_γ	
	√	√	√
	7	7	8
•PI_3 γ			×
PI_7 $\beta\gamma$		×	
PI_9 α	×		
PI_{11} $\beta\gamma$			×
•PI_{13} $\alpha\beta\gamma$	×	×	

Quine-McCluskey Tabular Minimization Method

$$f_\alpha = \mathrm{PI}_2 + \mathrm{PI}_5 + \mathrm{PI}_{13}$$
$$f_\beta = \mathrm{PI}_1 + \mathrm{PI}_5$$
$$f_\gamma = \mathrm{PI}_2 + \mathrm{PI}_3 + \mathrm{PI}_{13}$$

$$f_\alpha = \bar{A}\bar{B}\bar{D} + \bar{B}C\bar{D} + \bar{A}BCD$$
$$f_\beta = \bar{A}B + \bar{B}C\bar{D}$$
$$f_\gamma = \bar{A}\bar{B}\bar{D} + \bar{B}\bar{C}\bar{D} + \bar{A}BCD$$