# Algebraic Methods for the Analysis and Synthesis of Logic Circuit



# Agenda

- Fundamentals of Boolean Algebra
- Switching Function
- Switching Circuit
- Analysis of Combinational Circuit
- Synthesis of Combinational Logic Circuit

# Fundamentals of Boolean Algebra

- Basic Postulates
  - Postulate 1: Definition. A Boolean algebra is a closes algebraic system and 2 operators . and +;

    for every a and b in set K

    a.b belongs to K and a+b belongs to K

  - Postulate 2: Existence of 1 and 0 elements

    (a) a+0 = a

    (b) a.1 = a



# Fundamentals of Boolean Algebra

- Postulate 3: Commutativity of the + and . Operations.

  (a) a+b = b+a

  (b) a.b = b.a

- Postulate 4: Associativity of the + and . Operations.

  (a) a+(b+c) = (a+b)+c

  (b) a.(b.c) = (a.b).c

## Fundamentals of Boolean Algebra

- Postulate 5: Distributivity of + over . and . over +
  - (a) a+(b.c) = (a+b).(a+c)
  - (b) a.(b+c) = (a.b)+(a.c)

- Postulate 6: Existence of the complement
  - (a) a+(~a) = 1
  - (b) a.(~a) = 0



## Fundamentals of Boolean Algebra

- Fundamental Theorems of Boolean Algebra
  - Theorem 1. Idempotency
    - (a) a+a = a
    - (b) a.a = a

# Fundamentals of Boolean Algebra

- Theorem 2. Null elements for + and . operators

  (a) a+1 = 1

  (b) a.0 = 0

$$
\begin{aligned}
a + 1 &= (a + 1)1 && [\text{P2(b)}] \\
&= 1 \cdot (a + 1) && [\text{P3(b)}] \\
&= (a + \bar{a})(a + 1) && [\text{P6(a)}] \\
&= a + \bar{a} \cdot 1 && [\text{P5(a)}] \\
&= a + \bar{a} && [\text{P2(b)}] \\
&= 1 && [\text{P6(a)}]
\end{aligned}
$$



# Fundamentals of Boolean Algebra

- Theorem 3. Involution

  ~(~a) = a

- Theorem 4. Absorption

  (a) a+ab = a

  (b) a.(a+b) = a

$$
\begin{aligned}
a + ab &= a \cdot 1 + ab && [\text{P2(b)}] \\
&= a(1 + b) && [\text{P5(b)}] \\
&= a(b + 1) && [\text{P3(b)}] \\
&= a \cdot 1 && [\text{T2(a)}] \\
&= a && [\text{P2(b)}]
\end{aligned}
$$

$$(X + Y) + (X + Y)Z = X + Y \qquad [\text{T4(a)}]$$

$$A\bar{B}(A\bar{B} + \bar{B}C) = A\bar{B} \qquad [\text{T4(b)}]$$

$$A\bar{B}C + \bar{B} = \bar{B} \qquad [\text{T4(a)}]$$

# Fundamentals of Boolean Algebra

- Theorem 5.

**(a)** $a + \bar{a}b = a + b.$

**(b)** $a(\bar{a} + b) = ab.$

***Proof.*** Part (a) of the theorem is proved as follows:

$$\begin{aligned} a + \bar{a}b &= (a + \bar{a})(a + b) && [\text{P5(a)}] \\ &= 1 \cdot (a + b) && [\text{P6(a)}] \\ &= (a + b) \cdot 1 && [\text{P3(b)}] \\ &= (a + b) && [\text{P2(b)}] \end{aligned}$$

$B + A\bar{B}\bar{C}D = B + A\bar{C}D$ [T5(a)]

$\bar{Y}(X + Y + Z) = \bar{Y}(X + Z)$ [T5(b)]

$(X + Y)(\overline{(X + Y)} + Z) = (X + Y)Z$ [T5(b)]

$AB + \overline{(AB)}C\bar{D} = AB + C\bar{D}$ [T5(a)]



# Fundamentals of Boolean Algebra

- Theorem 6.

**(a)** $ab + a\bar{b} = a.$

**(b)** $(a + b)(a + \bar{b}) = a.$

***Proof.*** Part (a) of the theorem is proved as follows:

$$\begin{aligned} ab + a\bar{b} &= a(b + \bar{b}) && [\text{P5(b)}] \\ &= a \cdot 1 && [\text{P6(a)}] \\ &= a && [\text{P2(b)}] \end{aligned}$$

$ABC + A\bar{B}C = AC$ [T6(a)]

# Fundamentals of Boolean Algebra

- Theorem 7.

(a) $ab + a\bar{b}c = ab + ac$.

(b) $(a + b)(a + \bar{b} + c) = (a + b)(a + c)$.

*Proof.* Part (a) of the theorem is proved as follows:

$$\begin{aligned} ab + a\bar{b}c &= a(b + \bar{b}c) && \text{[P5(b)]} \\ &= a(b + c) && \text{[T5(a)]} \\ &= ab + ac && \text{[P5(b)]} \end{aligned}$$

$$xy + x\bar{y}(\bar{w} + \bar{z}) = xy + x(\bar{w} + \bar{z}) \qquad \text{[T7(a)]}$$

$$(\bar{x}\bar{y} + z)(w + \bar{x}\bar{y} + \bar{z}) = (\bar{x}\bar{y} + z)(w + \bar{x}\bar{y}) \qquad \text{[T7(b)]}$$



# Fundamentals of Boolean Algebra

$$\begin{aligned} (\bar{A} + \bar{B} + \bar{C})(\bar{B} + C)(A + \bar{B}) &= (\bar{A} + \bar{B})(\bar{B} + C)(A + \bar{B}) && \text{[T7(b)]} \\ &= \bar{B}(\bar{B} + C) && \text{[T6(b)]} \\ &= \bar{B} && \text{[T4(b)]} \end{aligned}$$

$$\begin{aligned} w\bar{y} + w\bar{x}y + wxyz + wx\bar{z} &= w\bar{y} + w\bar{x}y + wxy + wx\bar{z} && \text{[T7(a)]} \\ &= w\bar{y} + wy + wx\bar{z} && \text{[T6(a)]} \\ &= w + wx\bar{z} && \text{[T6(a)]} \\ &= w && \text{[T4(a)]} \end{aligned}$$

# Fundamentals of Boolean Algebra

- Theorem 8. DeMorgan's theorem

(a) $\overline{a + b} = \bar{a} \cdot \bar{b}$.
(b) $\overline{a \cdot b} = \bar{a} + \bar{b}$.

Theorem 8 may be generalized as follows.

(a) $\overline{a + b + \cdots + z} = \bar{a} \cdot \bar{b} \cdot \cdots \cdot \bar{z}$.
(b) $\overline{ab \ldots z} = \bar{a} + \bar{b} + \cdots + \bar{z}$.

**Complement the expression $a + bc$.**

$$\begin{aligned}\overline{a + b \cdot c} &= \overline{a + (b \cdot c)} \\ &= \bar{a} \cdot \overline{(b \cdot c)} \\ &= \bar{a} \cdot (\bar{b} + \bar{c}) \\ &= \bar{a}\bar{b} + \bar{a}\bar{c}\end{aligned}$$

Note that: $\overline{a + b \cdot c} \neq \bar{a} \cdot \bar{b} + \bar{c}$



# Fundamentals of Boolean Algebra

**Complement the expression $a(b + z(x + \bar{a}))$, and simplify the result so that the only complemented terms are individual variables.**

$$\begin{aligned}\overline{a(b + z(x + \bar{a}))} &= \bar{a} + \overline{(b + z(x + \bar{a}))} && \text{[T8(b)]} \\ &= \bar{a} + \bar{b}\,\overline{(z(x + \bar{a}))} && \text{[T8(a)]} \\ &= \bar{a} + \bar{b}(\bar{z} + \overline{(x + \bar{a})}) && \text{[T8(b)]} \\ &= \bar{a} + \bar{b}(\bar{z} + \bar{x} \cdot \bar{\bar{a}}) && \text{[T8(a)]} \\ &= \bar{a} + \bar{b}(\bar{z} + \bar{x}a) && \text{[T3]} \\ &= \bar{a} + \bar{b}(\bar{z} + \bar{x}) && \text{[T5(a)]}\end{aligned}$$

# Fundamentals of Boolean Algebra

**Repeat**

$a(b+c)+\bar{a}b.$

$$
\begin{aligned}
\overline{a(b+c)+\bar{a}b} &= \overline{ab+ac+\bar{a}b} && \text{[P5(b)]} \\
&= \overline{b+ac} && \text{[T6(a)]} \\
&= \bar{b}(\overline{ac}) && \text{[T8(a)]} \\
&= \bar{b}(\bar{a}+\bar{c}) && \text{[T8(b)]}
\end{aligned}
$$



# Fundamentals of Boolean Algebra

- Theorem 9. Consensus

(a) $ab+\bar{a}c+bc = ab+\bar{a}c.$

(b) $(a+b)(\bar{a}+c)(b+c) = (a+b)(\bar{a}+c).$

$$
\begin{aligned}
ab+\bar{a}c+bc &= ab+\bar{a}c+1\cdot bc && \text{[P2(b)]} \\
&= ab+\bar{a}c+(a+\bar{a})bc && \text{[P6(a)]} \\
&= ab+\bar{a}c+abc+\bar{a}bc && \text{[P5(b)]} \\
&= (ab+abc)+(\bar{a}c+\bar{a}cb) && \\
&= ab+\bar{a}c && \text{[T4(a)]}
\end{aligned}
$$

# Fundamentals of Boolean Algebra

$$AB + \bar{A}CD + BCD = AB + \bar{A}CD \qquad \text{[T9(a)]}$$

$$(a + \bar{b})(\bar{a} + c)(\bar{b} + c) = (a + \bar{b})(\bar{a} + c) \qquad \text{[T9(b)]}$$

$$\begin{aligned} ABC + \bar{A}D + \bar{B}D + CD &= ABC + (\bar{A} + \bar{B})D + CD \\ &= ABC + \overline{AB}D + CD \\ &= ABC + \overline{AB}D \\ &= ABC + (\bar{A} + \bar{B})D \\ &= ABC + \bar{A}D + \bar{B}D \end{aligned}$$



# Fundamentals of Boolean Algebra

| Expression | Dual |
|---|---|
| $P2(a): a + 0 = a$ | $P2(b): a \cdot 1 = a$ |
| $P3(a): a + b = b + a$ | $P3(b): ab = ba$ |
| $P4(a): a + (b + c) = (a + b) + c$ | $P4(b): a(bc) = (ab)c$ |
| $P5(a): a + bc = (a + b)(a + c)$ | $P5(b): a(b + c) = ab + ac$ |
| $P6(a): a + \bar{a} = 1$ | $P6(b): a \cdot \bar{a} = 0$ |
| $T1(a): a + a = a$ | $T1(b): a \cdot a = a$ |
| $T2(a): a + 1 = 1$ | $T2(b): a \cdot 0 = 0$ |
| $T3: \quad \bar{\bar{a}} = a$ | |
| $T4(a): a + ab = a$ | $T4(b): a(a + b) = a$ |
| $T5(a): a + \bar{a}b = a + b$ | $T5(b): a(\bar{a} + b) = ab$ |
| $T6(a): ab + a\bar{b} = a$ | $T6(b): (a + b)(a + \bar{b}) = a$ |
| $T7(a): ab + a\bar{b}c = ab + ac$ | $T7(b): (a + b)(a + \bar{b} + c) = (a + b)(a + c)$ |
| $T8(a): \overline{a + b} = \bar{a}\bar{b}$ | $T8(b): \overline{ab} = \bar{a} + \bar{b}$ |
| $T9(a): ab + \bar{a}c + bc = ab + \bar{a}c$ | $T9(b): (a + b)(\bar{a} + c)(b + c) = (a + b)(\bar{a} + c)$ |

# Switching function

- The results are valid for any Boolean algebra is often referred to as switching algebra.
- Since there are *n* variables, there are $2^n$ ways of assigning these values to the *n* variables.
- f(x1,x2,...,Xn) there are $2^y$ (when y = $2^n$) different switching functions of *n* variables.

For $n = 1$, the four functions of the variable $A$ are

$$f_0 = 0, \qquad f_2 = A$$
$$f_1 = \bar{A}, \qquad f_3 = 1$$



# Switching function

- The 16 functions of the two variables A and B.

$$f_0(A, B) = 0$$
$$f_1(A, B) = \bar{A}\bar{B}$$
$$f_2(A, B) = \bar{A}B$$
$$f_3(A, B) = \bar{A}B + \bar{A}\bar{B} = \bar{A}$$
$$f_4(A, B) = A\bar{B}$$
$$f_5(A, B) = A\bar{B} + \bar{A}\bar{B} = \bar{B}$$
$$f_6(A, B) = A\bar{B} + \bar{A}B$$
$$f_7(A, B) = A\bar{B} + \bar{A}B + \bar{A}\bar{B} = \bar{A} + \bar{B}$$
$$f_8(A, B) = AB$$
$$f_9(A, B) = AB + \bar{A}\bar{B}$$
$$f_{10}(A, B) = AB + \bar{A}B = B$$
$$f_{11}(A, B) = AB + \bar{A}B + \bar{A}\bar{B} = \bar{A} + B$$
$$f_{12}(A, B) = AB + A\bar{B} = A$$
$$f_{13}(A, B) = AB + A\bar{B} + \bar{A}\bar{B} = A + \bar{B}$$
$$f_{14}(A, B) = AB + A\bar{B} + \bar{A}B = A + B$$
$$f_{15}(A, B) = AB + A\bar{B} + \bar{A}B + \bar{A}\bar{B} = 1$$

| AB | $f_0$ | $f_1$ | $f_2$ | $f_3$ | $f_4$ | $f_5$ | $f_6$ | $f_7$ | $f_8$ | $f_9$ | $f_{10}$ | $f_{11}$ | $f_{12}$ | $f_{13}$ | $f_{14}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 00 | 0 | 1 | 0 | 1 | 0 | 1 | 0 | 1 | 0 | 1 | 0 | 1 | 0 | 1 | 0 |
| 01 | 0 | 0 | 1 | 1 | 0 | 0 | 1 | 1 | 0 | 0 | 1 | 1 | 0 | 0 | 1 |
| 10 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 1 | 1 | 1 |
| 11 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |

# Switching function

- Truth Table

| $a\ b$ | $f(a,b) = a + b$ |
|---|---|
| 0 0 | 0 |
| 0 1 | 1 |
| 1 0 | 1 |
| 1 1 | 1 |

| $a\ b$ | $f(a,b) = a \cdot b$ |
|---|---|
| 0 0 | 0 |
| 0 1 | 0 |
| 1 1 | 0 |
| 1 0 | 1 |

| $a$ | $f(a) = \bar{a}$ |
|---|---|
| 0 | 1 |
| 1 | 0 |

$$f(A, B, C) = AB + \bar{A}C + A\bar{C}$$

| $ABC$ | $f(A, B, C)$ | $ABC$ | $f(A, B, C)$ |
|---|---|---|---|
| 000 | 0 | $FFF$ | $F$ |
| 001 | 1 | $FFT$ | $T$ |
| 010 | 0 | $FTF$ | $F$ |
| 011 | 1 | $FTT$ | $T$ |
| 100 | 1 | $TFF$ | $T$ |
| 101 | 0 | $TFT$ | $F$ |
| 110 | 1 | $TTF$ | $T$ |
| 111 | 1 | $TTT$ | $T$ |



# Switching function

$$f(A, B, C) = AB + \bar{A}C + A\bar{C}$$

| $A, B, C$ | $AB$ | $\bar{A}$ | $\bar{A}C$ | $AB + \bar{A}C$ | $\bar{C}$ | $A\bar{C}$ | $(AB + \bar{A}C) + A\bar{C}$ |
|---|---|---|---|---|---|---|---|
| 000 | $0 \cdot 0 = 0$ | $\bar{0} = 1$ | $1 \cdot 0 = 0$ | $0 + 0 = 0$ | $\bar{0} = 1$ | $0 \cdot 1 = 0$ | $0 + 0 = 0$ |
| 001 | $0 \cdot 0 = 0$ | $\bar{0} = 1$ | $1 \cdot 1 = 1$ | $0 + 1 = 1$ | $\bar{1} = 0$ | $0 \cdot 0 = 0$ | $1 + 0 = 1$ |
| 010 | $0 \cdot 1 = 0$ | $\bar{0} = 1$ | $1 \cdot 0 = 0$ | $0 + 0 = 0$ | $\bar{0} = 1$ | $0 \cdot 1 = 0$ | $0 + 0 = 0$ |
| 011 | $0 \cdot 1 = 0$ | $\bar{0} = 1$ | $1 \cdot 1 = 1$ | $0 + 1 = 1$ | $\bar{1} = 0$ | $0 \cdot 0 = 0$ | $1 + 0 = 1$ |
| 100 | $1 \cdot 0 = 0$ | $\bar{1} = 0$ | $0 \cdot 0 = 0$ | $0 + 0 = 0$ | $\bar{0} = 1$ | $1 \cdot 1 = 1$ | $0 + 1 = 1$ |
| 101 | $1 \cdot 0 = 0$ | $\bar{1} = 0$ | $0 \cdot 1 = 0$ | $0 + 0 = 0$ | $\bar{1} = 0$ | $1 \cdot 0 = 0$ | $0 + 0 = 0$ |
| 110 | $1 \cdot 1 = 1$ | $\bar{1} = 0$ | $0 \cdot 0 = 0$ | $1 + 0 = 1$ | $\bar{0} = 1$ | $1 \cdot 1 = 1$ | $1 + 1 = 1$ |
| 111 | $1 \cdot 1 = 1$ | $\bar{1} = 0$ | $0 \cdot 1 = 0$ | $1 + 0 = 1$ | $\bar{1} = 0$ | $1 \cdot 0 = 0$ | $1 + 0 = 1$ |

# Switching function

- Algebraic Forms of switching function
  - SOP Forms : Sum of products

    - SOP are constructed by summing (ORing) product (ANDed) terms.

    - Each product term is formed by ANDing a number of complemented and un-complemented variables.

    - Each variables called *literal*.

$$f(A, B, C, D) = A\bar{B}C + \bar{B}\bar{D} + \bar{A}C\bar{D}$$



# Switching function

- POS Forms : Product of Sums

  - POS are constructed by taking the product (ANDing) of sum (ORed) terms.

  - Each product term is formed by ORing a number of literal.

$$f(A, B, C, D) = (\bar{A} + B + C)(\bar{B} + C + \bar{D})$$

## Switching function

- Canonical Forms:

  \- Minterm : The product term is call *minterm*.

  If the function is represented as a sum of minterm only, the function called *canonical sum of products* (canonical SOP) form.

$$f_\alpha(A, B, C) = \bar{A}B\bar{C} + AB\bar{C} + \bar{A}BC + ABC$$

Each minterm is represented by an n-bit binary code



## Switching function

Each bit represent one of the variables of minterm as follows:

Uncomplemented variable: 1
Complemented variable: 0

$$f_\alpha(A, B, C) = \bar{A}B\bar{C} + AB\bar{C} + \bar{A}BC + ABC$$

| Minterm | Minterm Code | Minterm Number |
|---|---|---|
| $\bar{A}B\bar{C}$ | 010 | $m_2$ |
| $AB\bar{C}$ | 110 | $m_6$ |
| $\bar{A}BC$ | 011 | $m_3$ |
| $ABC$ | 111 | $m_7$ |

$$f_\alpha(A, B, C) = m_2 + m_3 + m_6 + m_7$$

$$f_\alpha(A, B, C) = \sum m(2, 3, 6, 7)$$

# Switching function

The order of the variables in the functional notation is very important, since it determines the order of the bits of the minterm numbers.

$$
\begin{aligned}
f_\beta(B, C, A) &= \sum m(2, 3, 6, 7) \\
&= \underbrace{m_2}_{010} + \underbrace{m_3}_{011} + \underbrace{m_6}_{110} + \underbrace{m_7}_{111} \\
&= \bar{B}C\bar{A} + \bar{B}CA + BC\bar{A} + BCA \\
&= \bar{A}\bar{B}C + A\bar{B}C + \bar{A}BC + ABC
\end{aligned}
$$

$$
\begin{aligned}
f_\beta(A, B, C) &= f_\beta(B, C, A) \\
&= \underbrace{\bar{A}\bar{B}C}_{001} + \underbrace{\bar{A}BC}_{011} + \underbrace{A\bar{B}C}_{101} + \underbrace{ABC}_{111} \\
&= m_1 + m_3 + m_5 + m_7 \\
&= \sum m(1, 3, 5, 7)
\end{aligned}
$$



# Switching function

| Row No. ($i$) | **Inputs** $ABC$ | $m_1$ $\bar{A}\bar{B}C$ | $m_3$ $\bar{A}BC$ | $m_5$ $A\bar{B}C$ | $m_7$ $ABC$ | **Outputs** $f_\beta(A, B, C$ |
|---|---|---|---|---|---|---|
| 0 | 000 | 0 | 0 | 0 | 0 | 0 |
| 1 | 001 | 1 | 0 | 0 | 0 | 1 |
| 2 | 010 | 0 | 0 | 0 | 0 | 0 |
| 3 | 011 | 0 | 1 | 0 | 0 | 1 |
| 4 | 100 | 0 | 0 | 0 | 0 | 0 |
| 5 | 101 | 0 | 0 | 1 | 0 | 1 |
| 6 | 110 | 0 | 0 | 0 | 0 | 0 |
| 7 | 111 | 0 | 0 | 0 | 1 | 1 |

# Switching function

| Row No. ($i$) | Inputs $ABC$ | Outputs $f_\alpha(A, B, C)$ | | Complement $\bar{f}_\alpha(A, B, C)$ | |
|---|---|---|---|---|---|
| 0 | 000 | 0 | | 1 | $\leftarrow m_0$ |
| 1 | 001 | 0 | | 1 | $\leftarrow m_1$ |
| 2 | 010 | 1 | $\leftarrow m_2$ | 0 | |
| 3 | 011 | 1 | $\leftarrow m_3$ | 0 | |
| 4 | 100 | 0 | | 1 | $\leftarrow m_4$ |
| 5 | 101 | 0 | | 1 | $\leftarrow m_5$ |
| 6 | 110 | 1 | $\leftarrow m_6$ | 0 | |
| 7 | 111 | 1 | $\leftarrow m_7$ | 0 | |

$$f_\alpha(A, B, C) = \sum m(2, 3, 6, 7)$$

$$\bar{f}_\alpha(A, B, C) = \sum m(0, 1, 4, 5)$$



# Switching function

Given the function
$f(A,B,Q,Z) = \bar{A}\bar{B}\bar{Q}\bar{Z} + \bar{A}\bar{B}\bar{Q}Z + \bar{A}BQ\bar{Z} + \bar{A}BQZ$,
let us express the functions $f(A,B,Q,Z)$ and
$\bar{f}(A,B,Q,Z)$ in minterm list form.

$$\begin{aligned} f(A, B, Q, Z) &= \bar{A}\bar{B}\bar{Q}\bar{Z} + \bar{A}\bar{B}\bar{Q}Z + \bar{A}BQ\bar{Z} + \bar{A}BQZ \\ &= m_0 + m_1 + m_6 + m_7 \\ &= \sum m(0, 1, 6, 7) \end{aligned}$$

$\bar{f}(A, B, Q, Z)$ will contain the remaining 12 $(2^4 - 4)$ minterms. The minterm list for this function is

$$\begin{aligned} \bar{f}(A, B, Q, Z) &= m_2 + m_3 + m_4 + m_5 + m_8 + m_9 \\ &\quad + m_{10} + m_{11} + m_{12} + m_{13} + m_{14} + m_{15} \\ &= \sum m(2, 3, 4, 5, 8, 9, 10, 11, 12, 13, 14, 15) \end{aligned}$$

# Switching function

- Maxterm : The sum term is call *maxterm*.

If the function is represented as a product of minterm only, the function called *canonical product of sum* (canonical POS) form.

$$f_\gamma(A, B, C) = (A + B + C)(A + B + \bar{C})(\bar{A} + B + C)$$

Uncomplemented variable: 0
Complemented variable: 1



# Switching function

Uncomplemented variable: 0
Complemented variable: 1

| Maxterm | Maxterm Code | Maxterm List |
|---|---|---|
| $A + B + C$ | 000 | $M_0$ |
| $A + B + \bar{C}$ | 001 | $M_1$ |
| $\bar{A} + B + C$ | 100 | $M_4$ |
| $\bar{A} + B + \bar{C}$ | 101 | $M_5$ |

$$f_\gamma(A, B, C) = M_0 M_1 M_4 M_5 = \prod M(0, 1, 4, 5)$$

## Switching function

| Row No. $(i)$ | Inputs $ABC$ | $M_0$ $A+B+C$ | $M_1$ $A+B+\bar{C}$ | $M_4$ $\bar{A}+B+C$ | $M_5$ $\bar{A}+B+\bar{C}$ |
|---|---|---|---|---|---|
| 0 | 000 | 0 | 1 | 1 | 1 |
| 1 | 001 | 1 | 0 | 1 | 1 |
| 2 | 010 | 1 | 1 | 1 | 1 |
| 3 | 011 | 1 | 1 | 1 | 1 |
| 4 | 100 | 1 | 1 | 0 | 1 |
| 5 | 101 | 1 | 1 | 1 | 0 |
| 6 | 110 | 1 | 1 | 1 | 1 |
| 7 | 111 | 1 | 1 | 1 | 1 |

$$\begin{aligned} f_u(A, B, C) &= \sum m(2, 3, 6, 7) \\ &= f_\gamma(A, B, C) \\ &= \prod M(0, 1, 4, 5) \end{aligned}$$



## Switching function

**Given the function $f(A,B,C) =$ $(A+B+\bar{C})(A+\bar{B}+\bar{C})(\bar{A}+B+\bar{C})(\bar{A}+\bar{B}+\bar{C})$, let us construct the truth table and express the function in both maxterm and minterm form.**

$$f(A, B, C) = \underbrace{(A+B+\bar{C})}_{001}\underbrace{(A+\bar{B}+\bar{C})}_{011}\underbrace{(\bar{A}+B+\bar{C})}_{101}\underbrace{(\bar{A}+\bar{B}+\bar{C})}_{111}$$

$$= M_1 M_3 M_5 M_7$$

$$= \prod M(1, 3, 5, 7)$$

| Row No. $(i)$ | Inputs $ABC$ | Outputs $f(A, B, C)$ | $= \prod M(1, 3, 5, 7)$ |
|---|---|---|---|
| 0 | 000 | 1 | |
| 1 | 001 | 0 | $\leftarrow M_1$ |
| 2 | 010 | 1 | |
| 3 | 011 | 0 | $\leftarrow M_3$ |
| 4 | 100 | 1 | |
| 5 | 101 | 0 | $\leftarrow M_5$ |
| 6 | 110 | 1 | |
| 7 | 111 | 0 | $\leftarrow M_7$ |

# Switching function

From the truth table for $f(A, B, C)$, we observe that

$$f(A, B, C) = \sum m(0, 2, 4, 6)$$

Therefore,

$$\bar{f}(A, B, C) = \sum m(1, 3, 5, 7)$$
$$= \underbrace{m_1}_{001} + \underbrace{m_3}_{011} + \underbrace{m_5}_{101} + \underbrace{m_7}_{111}$$
$$= \bar{A}\bar{B}C + \bar{A}BC + A\bar{B}C + ABC$$

Consequently,

$$f(A, B, C) = \overline{\bar{A}\bar{B}C + \bar{A}BC + A\bar{B}C + ABC}$$
$$= \overline{\bar{A}\bar{B}C} \cdot \overline{\bar{A}BC} \cdot \overline{A\bar{B}C} \cdot \overline{ABC}$$
$$= \underbrace{(A + B + \bar{C})}_{001}\underbrace{(A + \bar{B} + \bar{C})}_{011}\underbrace{(\bar{A} + B + \bar{C})}_{101}\underbrace{(\bar{A} + \bar{B}}_{11}$$
$$= M_1 M_3 M_5 M_7$$
$$= \prod M(1, 3, 5, 7)$$

Therefore, we have algebraically shown that

$$f(A, B, C) = \prod M(1, 3, 5, 7) = \sum m(0, 2, 4, 6)$$



# Switching function

| Row No. $(i)$ | Inputs $ABC$ | Outputs $f(A, B, C)$ | Outputs $\bar{f}(A, B, C)$ | |
|---|---|---|---|---|
| 0 | 0 0 0 | 1 | 0 | $\leftarrow M_0$ |
| 1 | 0 0 1 | 0 | 1 | |
| 2 | 0 1 0 | 1 | 0 | $\leftarrow M_2$ |
| 3 | 0 1 1 | 0 | 1 | |
| 4 | 1 0 0 | 1 | 0 | $\leftarrow M_4$ |
| 5 | 1 0 1 | 0 | 1 | |
| 6 | 1 1 0 | 1 | 0 | $\leftarrow M_6$ |
| 7 | 1 1 1 | 0 | 1 | |

$$\bar{f}(A, B, C) = \prod M(0, 2, 4, 6)$$

$$f(A, B, C) = \prod M(1, 3, 5, 7)$$

$$f(A, B, C) = \sum m(0, 2, 4, 6) = \prod M(1, 3, 5, 7)$$

$$\bar{f}(A, B, C) = \sum m(1, 3, 5, 7) = \prod M(0, 2, 4, 6)$$

# Switching function

- Derivation of Canonical Forms

**Convert the following function to canonical SOP form:**

$$f(A,B,C) = AB + A\bar{C} + \bar{A}C$$

Let us apply Theorem 6a to each of the three product terms of this

$$AB = AB\bar{C} + ABC = m_6 + m_7$$
$$A\bar{C} = A\bar{C}\bar{B} + A\bar{C}B = A\bar{B}\bar{C} + AB\bar{C} = m_4 + m_6$$
$$\bar{A}C = \bar{A}C\bar{B} + \bar{A}CB = \bar{A}\bar{B}C + \bar{A}BC = m_1 + m_3$$

Therefore,

$$f(A, B, C) = AB + A\bar{C} + \bar{A}C$$
$$= (m_6 + m_7) + (m_4 + m_6) + (m_1 + m_3)$$
$$= \sum m(1, 3, 4, 6, 7)$$



# Switching function

**Expand the following function to canonical POS form:**

$$f(A,B,C) = A(A + \bar{C})$$

Theorem 6b can be applied as follows to produce maxterms.

$$A = (A + \bar{B})(A + B)$$
$$= (A + \bar{B} + \bar{C})(A + \bar{B} + C)(A + B + \bar{C})(A + B -$$
$$= M_3M_2M_1M_0$$
$$(A + \bar{C}) = (A + \bar{C} + \bar{B})(A + \bar{C} + B)$$
$$= (A + \bar{B} + \bar{C})(A + B + \bar{C})$$
$$= M_3M_1$$

Therefore,

$$A(A + C) = (M_3M_2M_1M_0)(M_3M_1)$$
$$= \prod M(0, 1, 2, 3)$$

# Switching function

- Incompletely Specified Function

  - The don't care minterms will be labeled $d_i$ instead of $m_i$

  - The don't care maxterms will be labeled $D_i$ instead of $M_i$



# Switching function

**Suppose that we are given a function $f(A,B,C)$ that has minterms $m_0$, $m_3$, and $m_7$ and don't-care conditions $d_4$ and $d_5$. We wish to express the function and its complement in both minterm and maxterm form and then reduce the function to its simplest form.**

The minterm list form for this function is

$$f(A, B, C) = \sum m(0, 3, 7) + d(4, 5)$$

and the maxterm list is

$$f(A, B, C) = \prod M(1, 2, 6) \cdot D(4, 5)$$

$$\bar{f}(A, B, C) = \sum m(1, 2, 6) + d(4, 5)$$
$$= \prod M(0, 3, 7) \cdot D(4, 5)$$

# Switching function

$$f(A, B, C) = \bar{A}\bar{B}\bar{C} + \bar{A}BC + ABC + d(A\bar{B}\bar{C} + A\bar{B}C$$

$$f(A, B, C) = \bar{A}\bar{B}\bar{C} + BC + d(A\bar{B}\bar{C} + A\bar{B}C)$$

$$\begin{aligned} f(A, B, C) &= \bar{A}\bar{B}\bar{C} + BC + A\bar{B}\bar{C} \\ &= \bar{B}\bar{C} + BC \end{aligned}$$